IEEM FFT—A Fast and Efficient Tool for Rigorous Computations of Propagation Constants and Field Distributions in Dielectric Guides with Arbitrary Cross-Section and Permittivity Profiles

Michał P. Mrozowski

Abstract—**An efficient approach to the analysis of dielectric guides is presented. The technique described is based on a recently proposed iterative scheme [5], [6] known as the iterative eigenfunction expansion method (IEEM), which was designed specifically to allow the rigorous analysis of dielectric guides of arbitrary cross section and permittivity profile. In the approach presented herein, which we shall call the IEEM FFT, the bottleneck of the IEEM is removed by the application of the FFT to the calculation of the inner product. As a result, a reduction in the computer storage and an increase in speed are achieved. In some aspects the method seems to be superior to certain full-wave approaches, including the finite difference and finite element methods. It is believed that the method can be used for investigating guides used in millimeter-wave techniques, optical fibers with arbitrary cross section and refractive index profiles, and nonlinear effects in electromagnetic wave propagation.**

I. Introduction

DIELECTRIC waveguides are of paramount importance in constructing millimeter-wave and integrated optics devices. As new manufacturing technologies develop, the range of options increases and the problem of characterizing new structures arises. For certain specific guide geometries there exists a choice of efficient methods which provide both the propagation constant and the field distribution. However, when it comes to the analysis of arbitrarily shaped dielectric guides which may be transversely inhomogeneous, there are only a few numerical methods which are capable of solving the wave equation effectively. These methods include such techniques as the coupled mode, the Galerkin, the finite element, and the finite difference [2]–[4], [8]–[9]. In 1985, in a survey paper devoted to the numerical methods for analyzing arbitrarily shaped microwave and optical guides [1], Saad noted that the numerical tools which existed at that time did not meet the needs of users as new practical waveguide possibilities emerged. One problem with such numerical methods as the finite difference [9] and the finite element [10] was that spurious eigenvalues occurred and special tests had to be used to identify these solutions. In the seminumerical approaches, such as the coupled mode [2] and the Galerkin [3], the computer storage requirements posed a problem as these methods involve the construction of dense matrices of large order. In conclusion it was pointed out that substantial new work was required to improve the existing options and broaden the range of available numerical tools by creating new. competitive methods or possibly better ones.

Since then the known methods have been improved (e.g., a formulation free of the spurious modes has been proposed [4], [16]–[20]) and new techniques, such as the integral domain method [12], have been developed. A salient feature of all these methods is that at some stage the boundary value problem is converted into matrix form. If n denotes the matrix order, then matrix-oriented methods require $\sim n^2$ memory locations and give the solution after $\sim n^3$ operations. The storage and computer time can be dramatically reduced if sparse matrix techniques can be used, but this is at the cost of increased complexity of the numerical implementation. Unfortunately, sparse matrix techniques have been used in practice only when the frequency was treated as an eigenvalue [8], [9]. This approach has certain drawbacks [17] and in recent formulations of the FEM and FDM [4], [16]–[20] the propagation constant is usually treated as an eigenvalue and dense solvers are used.

In this paper we propose a technique which consists in an alternative realization of the numerical algorithm of the recently developed [5], [6] iterative eigenfunction expansion method (IEEM). The IEEM was designed to analyze arbitrarily shaped, transversely inhomogeneous dielectric guides. The IEEM is fast, does not require any matrix computation, and provides the propagation constant and the field distribution simultaneously. In our approach we use the FFT to compute the scalar products in the iteration loop of the IEEM. When the IEEM is combined with the FFT, an increase in speed and a decrease in computer storage are achieved, resulting in an algorithm inheriting the virtues of the IEEM and requiring $O(n \log_2 n)$ operations and $O(n)$ computer locations, with n denoting the number of variables used in the approximation of the unknown fields.

Manuscript received April 9, 1990; revised September 5, 1990.

The author is with the Telecommunication Institute, Technical University of Gdańsk, 80-952 Gdańsk, Poland.

IEEE Log Number 9041091.

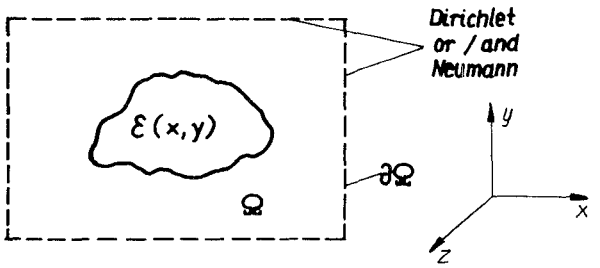


Fig. 1. Cross section of the dielectric guide under investigation. The screening walls may be real or imaginary for open guides or symmetrical structures.

II. Analysis

We shall investigate uniform dielectric isotropic guides of arbitrary cross section and arbitrary variation of the permittivity in both transverse directions (Fig. 1). For the sake of analysis, we assume that the guide is bounded by screening walls which may be located sufficiently far away from the guide. The boundary conditions at the screening walls are assumed to be homogeneous Dirichlet and/or Neumann ones. For reasons which will be apparent later, we assume that the region Ω bounded by the screening walls is rectangular. The relative permittivity of the medium within the bounded region is given by a scalar function $\epsilon(x, y)$ (possibly complex) of both transverse coordinates.

The electromagnetic wave propagation in the z direction in such a structure is governed by the following boundary value problem (the field dependence is assumed to be of the form $e^{j(\omega t-\beta z)}$):

$$\nabla_t^2\vec{H}_t + k_0^2\epsilon(x,y)\vec{H}_t + \frac{1}{\epsilon(x,y)}\left[\nabla_t\epsilon(x,y)\times\left(\nabla_t\times\vec{H}_t\right)\right] - \beta^2\vec{H}_t = 0 \quad (1a)$$

$$\boldsymbol{B}\vec{H}_t = 0 \qquad \text{on } \partial\Omega \quad (1b)$$

where $\vec{H}_t$, k_0, and β are the transverse magnetic filed, the wavenumber in the free space, and an unknown propagation constant, respectively. $\boldsymbol{B}$ denotes a boundary condition operator. The H_t formulation was chosen since the magnetic field is continuous across the interfaces between nonmagnetic materials.

In general, an analytic solution to the above equation cannot be found and one has to resort to numerical methods. In this paper we shall apply for this purpose the IEEM. This method was developed in 1986 by Jabłoński [5], [10] and used for the investigation of multicore optical fibers [6].

III. The IEEM

The IEEM can be presented as follows: Suppose that we have to solve the eigenproblem

$$\boldsymbol{T}u - \lambda u = 0 \quad (2)$$

where u and λ denote the unknown eigenfunction and eigenvalue of operator $\boldsymbol{T}$, respectively. Roughly speaking, the above problem can be solved with the IEEM when it is possible to decompose the operator $\boldsymbol{T}$ so that $\boldsymbol{T} = \boldsymbol{L} - \boldsymbol{F}$, where $\boldsymbol{L}$ has to be a self-adjoint operator of a known set of eigenvalues $\{\Lambda_t\}$ and eigenfunctions $\{f_t\}$. The operator $\boldsymbol{F}$ describes the perturbation (not necessarily small) and should be relatively compact with respect to $\boldsymbol{L}$. The iteration loop of the IEEM is then

A) Initial approximation of u_0, λ_0 ($\boldsymbol{F}u_0 \neq 0$ and $\lambda_0 \neq \Lambda_t$).
Step k

B) Expansion of the kth approximation of u into series of eigenfunctions f_i:

$$\tilde{u}^{(k)} = \sum_t c_t^{(k)} f_t \quad (3)$$

The expansion coefficients are given by

$$c_t^{(k)} = \frac{\langle \boldsymbol{F}u^{(k-1)}, f_t\rangle}{\Lambda_i - \lambda^{(k-1)}} \quad (4)$$

with $\langle\cdot,\cdot\rangle$ denoting inner product.

C) Normalization of $u^{(k)}$:

$$u^{(k)} = \sum_i d_t^{(k)} f_t \quad (5)$$

where

$$d_t^{(k)} = \frac{c_t^{(k)}}{\sqrt{\sum_t |c_t^{(k)}|^2}}. \quad (6)$$

D) Determination of the kth approximation of the eigenvalue λ:

$$\lambda^{(k)} = \sum_t \Lambda_t |d_t^{(k)}|^2 - \sum_t d_i^{*(k)}\langle \boldsymbol{F}u^{(k)}, f_t\rangle. \quad (7)$$

Steps B, C, and D are repeated iteratively until

$$\left|\frac{\lambda^{(k)} - \lambda^{(k-1)}}{\lambda^{(k)}}\right| \leqslant \epsilon_\lambda \quad (8a)$$

and/or

$$\left|\|\tilde{u}^{(k)}\| - 1\right| \leqslant \epsilon_u \quad (8b)$$

where ϵ_λ and ϵ_u are understood as convergence thresholds.

In order to solve the boundary value problem (1) we put $\lambda = \beta^2$ and $u = \vec{H}_t = [H_x, H_y]$ and decompose the differential operator into

$$\boldsymbol{L} = \nabla_t^2 + k_0^2$$

$$\boldsymbol{F} = -(\epsilon(x,y)-1)k_0^2 - \frac{1}{\epsilon(x,y)}\left[\nabla_t\epsilon(x,y)\times(\nabla_t\times\cdot)\right]. \quad (9)$$

For the assumed homogeneous Dirichlet and/or Neumann boundary conditions, the operator $\boldsymbol{L}$ is self-adjoint. Moreover, it was shown in [10] that $\boldsymbol{F}$ is relatively compact with respect to $\boldsymbol{L}$.

IV. The IEEM FFT

The critical point in the IEEM is the calculation of the inner products appearing in (3). Until now the integrals involved have been computed either numerically [6] or analytically [7]. Integration has to be performed over the dielectric core; hence the analytical integration is possible only for certain core shapes. Numerical integration involves repeated calculation of transcendental functions, so unless the values of transcendental functions are stored in lookup tables, the evaluation of the inner product is very time consuming.

When the operator $\boldsymbol{L}$ is defined on a rectangle, the calculation of integrals and lookup tables for transcendental functions can be avoided altogether. In this case the set of eigenfunctions consists of sine and cosine functions. Consequently the coefficients c_i are obtained simply from the Fourier coefficients of the function $\boldsymbol{Fu}$. The fastest way to compute these coefficients regardless of the dielectric core shape and the permittivity profile is to apply the 2-D FFT. In order to use the FFT we have to know $\boldsymbol{Fu}$ in the spatial domain. This means that we have to move back and forth between the Fourier and spatial domains, first taking the 2-D inverse FFT to get u from the coefficients d_i known from a previous step, calculating $\boldsymbol{Fu}$, and then going back to the Fourier domain to get the coefficients c_i. More specifically, each step requires two inverse 2-D FFT's and two forward 2-D FFT's. Note that the operator $\boldsymbol{F}$ involves the derivation with respect to transverse coordinates. Since the inverse FFT gives the values of fields at discrete points, the derivatives can readily be found using a central difference scheme. (The derivatives can alternatively be computed in the Fourier domain and then calculated in the spatial domain by means of the inverse FFT. The numerical cost of this approach is slightly higher than in the case of central difference computations.)

Having presented the main idea underlying the IEEM FFT, we shall now discuss the algorithm in detail. Let us denote the eigenfunctions of $\boldsymbol{L} = \partial^2/\partial x^2 + \partial^2/\partial y^2 + k_0^2$ by h_x^i and h_y^i and the corresponding eigenvalues by Λ_x^i and Λ_y^i ($h_x^i = h_y^i$ and $\Lambda_x^i = \Lambda_y^i$ when we assume identical boundary conditions for both field components, which is a sensible assumption when open structures are to be modeled).

The iteration steps are as follows.

B.1) Using Fourier coefficients computed in the previous, $(k-1)$th step, evaluate $\vec{H}_t$ in the spatial domain:

$$H_x^{(k-1)} = \sum_i c_x^i h_x^i \qquad \text{2-D inverse FFT}$$

$$H_y^{(k-1)} = \sum_i c_y^i h_y^i \qquad \text{2-D inverse FFT.}$$

B.2) Compute $\nabla_t \times \vec{H}_t$, i.e., $\partial/\partial y H_x^{(k-1)}$ and $\partial/\partial x H_y^{(k-1)}$ (central differences).

B.3) Using data obtained in B.1 and B.2, compute $\boldsymbol{F}_x \vec{H}_t^{(k-1)}$ and $\boldsymbol{F}_y \vec{H}_t^{(k-1)}$, where

$$\boldsymbol{F}_x = -\left[(\epsilon(x,y)-1)k_0^2\right]_x - \frac{1}{\epsilon(x,y)}\frac{\partial}{\partial y}(\epsilon(x,y)-1)\left[-\frac{\partial}{\partial y}(\cdot); \frac{\partial}{\partial x}(\cdot)\right]$$

$$\boldsymbol{F}_y = -\left[(\epsilon(x,y)-1)k_0^2\right]_y - \frac{1}{\epsilon(x,y)}\frac{\partial}{\partial x}(\epsilon(x,y)-1)\left[\frac{\partial}{\partial y}(\cdot); -\frac{\partial}{\partial x}(\cdot)\right].$$

B.4) Compute the inner products $\langle \boldsymbol{F}_x \vec{H}_t^{(k-1)}, h_x^i \rangle$ and $\langle \boldsymbol{F}_y \vec{H}_t^{(k-1)}, h_y^i \rangle$ (2-D forward FFT).

B.5) Compute expansion coefficients c_x^i and c_y^i:

$$c_x^i = \frac{\langle \boldsymbol{F}_x \vec{H}_t^{(k-1)}, h_x^i \rangle}{\Lambda_x^i - \beta^2} \quad \text{and} \quad c_y^i = \frac{\langle \boldsymbol{F}_y \vec{H}_t^{(k-1)}, h_y^i \rangle}{\Lambda_y^i - \beta^2}.$$

C) Normalize the expansion coefficients:

$$d_x^i = \frac{c_x^i}{\sqrt{\Sigma_i\left(|c_x^i|^2 + |c_y^i|^2\right)}} \quad \text{and} \quad d_y^i = \frac{c_y^i}{\sqrt{\Sigma_i\left(|c_x^i|^2 + |c_y^i|^2\right)}}$$

D) Determine the new, kth approximation of the propagation constant β^2:

$$\beta^2 = \sum_i \left(\Lambda_x^i |d_x^i|^2 + \Lambda_y^i |d_y^i|^2\right) - \sum_i \left(d_x^{i*} \langle \boldsymbol{F}_x \vec{H}_t^{(k)}, h_x^i \rangle + d_y^{i*} \langle \boldsymbol{F}_y \vec{H}_t^{(k)}, h_y^i \rangle\right).$$

A. Operation Count

The computational cost of one iteration depends on the length of the FFT, the number of eigenfunctions K_1 and K_2 used for the expansion of the H_x and H_y, and the number of discretization points M within the cross section of the guide core ($\boldsymbol{Fu}$ does not vanish only in the region for which $\epsilon(x,y)-1 \neq 0$). The length of the FFT in each direction determines the discretization step. The computational cost of an FFT of length N is of the order $N \log_2 N$ (or less if the Winograd algorithm [13] is used). Let us denote by N_x and N_y the numbers of Fourier samples in the x and y directions and assume that $K_1 = K_2 = K = K_x K_y$. Suppose that $M = M_x M_y$; then the computational cost of each iteration is

Steps B.1 and B.4	$4\left(K_x N_y \log_2 N_y + M_y N_x \log_2 N_x\right)$
Steps B.2 and B.3	$6M$
Steps B.5, C and D	$12K$.

In total, one iteration requires of order $4(K_x N_y \log_2 N_y + M_y N_x \log_2 N_x) + 6M + 12K$ multiplications.

B. Memory Requirements

An important point to note is that the IEEM FFT is extremely efficient in terms of computer storage. In contrast to other full-wave methods for analyzing dielectric guides, the boundary value problem is not converted in the IEEM to an equivalent matrix problem (e.g. by means of the Galerkin procedure). Consequently the IEEM requires far less computer storage than matrix-oriented methods. In the IEEM FFT, a further reduction in terms of memory can be obtained in that no lookup tables with the values of transcendental functions need to be stored. For instance, if K basis functions for each field component are used and the discretization step is chosen so that M points lie within the core, the IEEM FFT requires typically only $2K + 5M$ memory locations.

The memory requirements can be further reduced if the permittivity profile and its derivatives are not stored but rather are computed at each iteration. In this case the computational cost increases slightly but the method can be implemented using only $2(K+M)$ memory locations. When $K = K_x K_y < M \leq M_x M_y$ the IEEM FFT can be implemented using $2K + 2\max(M_x K_y, M_y K_x)$ elements.

Let conclude this section with a brief discussion of the price which is paid in introducing the FFT to the IEEM. First of all we lose the freedom of choice of the basis functions. As a result we will sometimes, e.g. in case of guides with circular symmetry, have to use more functions than for an optimally selected basis. Moreover, we restrict a class of solvable problems to structures which are bounded by a rectangle. Finally, we have to use a uniform discretization. In return, we obtain a cost reduction in both memory and time.

V. IEEM-FFT Versus Matrix-Oriented Methods

Let us first consider the memory requirements of the matrix-oriented methods. We shall restrict ourselves to two groups of methods. The first includes the coupled mode (CMM) and Galerkin (GM) methods, while the second comprises finite difference (FD) and finite element (FE) methods. In the first group the solution is constructed using basis functions similar to those in the IEEM FFT but the wave equation is converted into a matrix eigenvalue problem with a dense, nonsymmetric characteristic matrix. When K basis functions for each field component are used, the characteristic matrix is of order $2K$. Hence, the methods of the first group require at least $4K^2$ memory locations compared with the minimum of $2K+2\max(M_xK_y, M_yK_x)$ in the IEEM FFT.

In the methods belonging to the second group, the matrix dimensions depend on the mesh size. To compare these methods with the IEEM FFT, suppose that the grid has $N_x \times N_y$ points and $N_x = N_y = \sqrt{N}$. The FD method generates matrices of order $2(N_x-1)^2$ [9]. Depending on the formulation, the matrices are symmetric banded [9] or nonsymmetric banded [4], with the bandwidth of order $2N_x$. However, despite matrices being sparse, the numerical procedures used for solving the eigenproblems, contained in EISPACK [11], require a lot of work space. For instance procedures used by Schweig and Bridges work with at least $\sim 8(N_x-1)^2(N_x+1)+N_x$ elements. In the formulation which is free from spurious modes and leads to a nonsymmetric banded matrix, the QR procedures from EISPACK [11] recommended by Bierwirth *et al.* [4] do not take any advantage of the sparsity of the characteristic matrix and consequently require at least $8(N_x-1)^4+6(N_x-1)^2$ storage locations which for $N_x=15$ results in a 4 Mb program [16]! This value is in sharp contrast to the minimum of $2K+2\max(M_xKy, M_yK_x)$, $K < N_xN_y$, required by the IEEM FFT.

As for the FE method two approaches have to be considered because drastically different memory requirements are obtained depending on whether the frequency or the propagation constant is treated as an eigenvalue. If frequency is an eigenvalue, the FEM results in a generalized eigenvalue problem with two symmetric (or Hermitian) sparse banded matrices. The smallest eigenvalues correspond to the lowest order modes. For definite problems there are sparse solvers computing a few largest (smallest) eigenvalues. For a given region division, the number of nonzero elements depends on the formulation used and the order of shape functions. Let us assume that the number of nodal points is N_x^2 and the size of matrices involved is of order N_x^2 (it may be higher, e.g. from $3N_x^2$ to $6N_x^2$ [20]). The number of nonzero elements is then WN_x^2, where W is the bandwidth. As in the simplest case of scalar field formulation and first-degree polynomials, the bandwidth is 7 and increases if higher order shape functions and/or vector fields are used; we may assume that the memory requirements for storing the matrices alone exceed $7N_x^2$. In practice this value is certainly higher since vector field formulations are used and additional work space has to be provided for numerical procedures. The approach with frequency as an eigenvalue has certain limitations as the computations for complex propagation constant and frequency dispersive media are difficult. Note that β may be complex even in lossless isotropic guides (complex waves). Therefore in recent formulations of the FEM [17]–[20] β is usually treated as an eigenvalue. This approach results either in an indefinite and singular symmetric generalized eigenvalue problem or in a nonsymmetric standard eigenvalue problem. Some of the formulations give sparse matrices [19]; some do not [17], [18]. To the best of the author's knowledge, well-known software libraries do not contain sparse routines which would handle an indefinite and singular symmetric generalized eigenvalue problem or a nonsymmetric standard eigenvalue problem. Therefore, in practice the FEM is implemented using dense solvers [17]–[20]. The size of the matrices involved ranges from around $2N_x^2$ [19] to around $6N_x^2$ [17]. This means that the memory requirements for implementing the QR algorithm vary from $8N_x^4+6N_x^2$ to $72N_x^4+18N_x^2$ with a program size of around 27 Mb (!) for 153 nodal points [18].

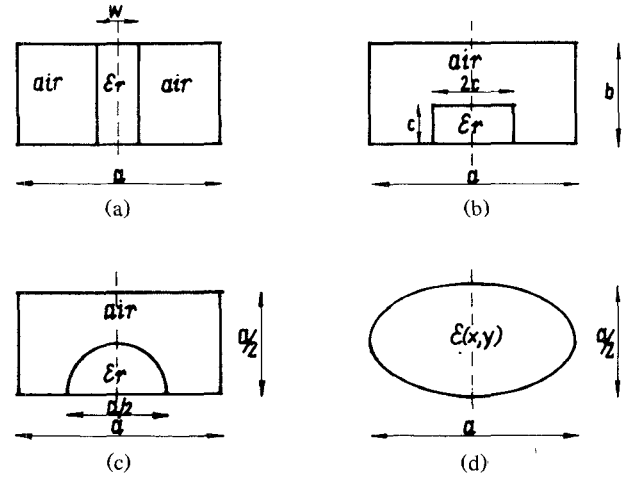


Fig. 2. Structures for which the test results obtained with the IEEM FET are compared with data available from the literature.

We shall conclude this section with a brief discussion of the numerical cost of matrix-oriented methods. We shall consider only the cost of numerical methods used to solve the matrix problem. In most cases, especially when the matrix is dense, the numerical cost of such methods is of order n^3, with n being the matrix order. This gives $\sim 8K^3$ for methods belonging to the first group (CMM and GM), $\sim 8(N_x-1)^6$ for the FD scheme, [4], [16] and from $\sim 8N_x^6$ to $\sim 216N_x^6$ for the FEM employing dense matrix solvers [17]–[20]. For sparse matrices arising in the FEM and FDM formulations in which the frequency is treated as an eigenvalue, this figure is obviously much lower but its actual value depends on the sparsity and the algorithm used. For instance, with the QL iterative algorithm for a tridiagonal symmetric matrix, which is a prototype of all sparse matrices, the work load for computing one eigenvalue is of order

TABLE I
NORMALIZED PROPAGATION CONSTANT β/k_0

	Mode TE^x_{10}			Mode TE^x_{20}		
Slab Width	TRM	IEEM FFT	Error	TRM	IEEM FFT	Error
0	0.87952	0.87952	0	0.30698	0.30698	0
1 mm	0.97173	0.97221	$+4.9\cdot10^{-4}$	0.30877	0.30880	$+6\cdot10^{-5}$
4 mm	1.23534	1.23528	$-4.8\cdot10^{-5}$	0.41570	0.41563	$-1.7\cdot10^{-4}$
12 mm	1.48695	1.48692	$-2\cdot10^{-5}$	1.10580	1.10565	$-1.3\cdot10^{-4}$
20 mm	1.52751	1.52751	0.	1.28574	1.28574	0.
21 mm	1.52760	1.52760	0.	1.28617	1.28617	0.

Slab guide (Fig. 2(a)), $\epsilon_r = 2.56$, $a = 21$ mm, $f = 15.0$ GHz.

TABLE II
NORMALIZED PROPAGATION CONSTANT β/k_0

	Mode E^y_{11}			Mode E^y_{21}	
CMM [2]	IEEM FFT	Error	CMM [2]	IEEM FFT	Error
1.529	1.534	$+3.2\cdot10^{-3}$	1.459	1.460	$+6.2\cdot10^{-4}$

Image guide (Fig. 2(b)) $\epsilon_r = 2.5$, $k_0c = 5$, $a/c = 5$, $b/c = 2.5$.

$\sim 20n$ per iteration [13]. The computation of eigenvectors, which is often needed for testing spurious modes and field computations, increases both numerical cost and the memory.

VI. VALIDATION OF THE IEEM FFT

The performance of the IEEM FFT can be improved by a suitable choice of the FFT algorithm. We chose the procedures from Swarztrauber's public domain FFTPACK library, which may be obtained via electronic mail [14]. This library uses the Winograd algorithm, which gives the user greater flexibility in the choice of the FFT length and can be up to two times faster than a classical base-2 Cooley–Tukey algorithm [13].

Numerous tests have been carried out in order to verify the IEEM FFT. First of all, it was verified that the solution obtained with the method is identical to the Galerkin solution of (1a) using the same set of basis functions. Fig. 2 shows structures for which the test results will be presented. All examples discussed were computed on 640 KB personal computer using a single FORTRAN program. The size of the program, which can handle around $M = 10^4$ discretization points within the guide core region and around $K = 10^4$ basis functions, was around 500 KB. In all examples the convergence thresholds were $\epsilon_\lambda = 10^{-5}$ and $\epsilon_u = 10^{-4}$. The field distribution of the fundamental mode in a homogeneous guide was chosen as a starting point for the iteration, but it was verified that convergence is also obtained from other initial solutions. Convergence was obtained typically after six to ten iterations. First the propagation constants of TE^x modes in a rectangular waveguide loaded with a dielectric slab (Fig. 2(a)) were computed.

The results are compared in Table I with the values obtained from the transverse resonance method (TRM). For convenience the relative error is also given. The agreement is excellent and for homogeneous or almost homogeneous guides the results are exact. Table II compares the IEEM FFT with the coupled mode method used by Ogusu to investigate a dielectric image guide [2] (Fig. 2(b)). In both methods the same number of basis functions, 2×49, was used. The relative error does not exceed 0.4%. Table III shows a comparison of the results of calculations with numerical and experimental data given by Baier [3], who used a direct Galerkin method to solve the E_t field wave equation. The structure under investigation was a resonator consisting of a section of a rectangular guide comprising a semicircular dielectric rod (Fig. 2(c)). The measurement error was estimated to be 1%. The results of the IEEM FFT lie well within the measurement error margin and for the quasi-TE_{20} mode are far more accurate than Baier's computations. Finally a comparison was made between the IEEM FFT and the original version of the IEEM (Table IV). Note that the IEEM as formulated in [6] uses a different basis than the IEEM FFT. While in the IEEM the fields are approximated by series of Bessel functions, the IEEM FFT uses trigonometric expansion. Also, the way the inner products are computed is entirely different. Computations were carried out for an inhomogeneous elliptical guide with semiaxes

TABLE III
MEASURED AND CALCULATED FREQUENCIES IN GHz

Mode	Measured [3]	IEEM FFT	Error	GM [3]
quasi TE_{10}	1.263	1.269	$+4.8\cdot10^{-3}$	1.267
quasi TE_{20}	1.672	1.678	$+3.6\cdot10^{-3}$	1.700

Rectangular guide with semicircular rod (Fig. 2(c)).
$\epsilon_r = 2.495$, resonator length $l = 11.1$ cm, $a = 20$ cm

TABLE IV
NORMALIZED PROPAGATION CONSTANT Z

α	IEEM [6], [15]	IEEM FFT	Error
2	0.48939	0.48948	$+1.8\cdot10^{-4}$
4	0.62536	0.62532	$-6.4\cdot10^{-5}$
6	0.67397	0.67383	$-2.1\cdot10^{-4}$
8	0.69760	0.69742	$-2.4\cdot10^{-4}$
10	0.71122	0.71098	$-3.4\cdot10^{-4}$

Elliptical guide, power function permittivity profile (Fig. 2(d)).
Normalized frequency $V = 3$, mode HE^y_{11}, $\epsilon = 2.25$, semiaxes ratio 2/1.

TABLE V
SPEED AND CONVERGENCE OF IEEM FFT

N_x	K_x	M_x	Z	Iterations	Time/Itr [s]
16	4	3	0.1375	5	0.22
16	8	3	0.2641	7	0.243
16	16	3	0.3126	10	0.285
32	4	5	0.1224	5	0.286
32	8	5	0.2392	7	0.292
32	16	5	0.2830	9	0.385
32	32	5	0.2842	8	0.625
64	4	8	0.1273	5	0.374
64	8	8	0.2525	7	0.376
64	16	8	0.29136	9	0.526
64	32	8	0.2925	8	0.852
64	64	8	0.2925	8	1.531
128	4	14	0.1279	5	0.67
128	8	14	0.2536	7	0.768
128	16	14	0.2912	9	1.
128	32	14	0.2925	8	1.56
128	64	14	0.2925	8	2.67
256	4	27	0.1276	5	1.9
256	8	27	0.2524	7	2.04
256	16	27	0.2898	9	2.454
256	32	27	0.29115	8	3.43
256	64	27	0.29115	8	5.38

Cylindrical guide, power function permittivity profile ($\alpha = 2$), mode HE_{11}^{y}.
$\epsilon = 2.25$, $V = 3$, $N_x = N_y$, $M_x = M_y$, $K_x = K_y$, screen at $10a_x$.

ratio $a_x/a_y = 2/1$ and the permittivity given by the function

$$\epsilon_r(x,y) = \epsilon\left\{1 - \left[(x/a_x)^2 + (y/a_y)^2\right]^{\alpha/2}\right\}. \quad (10)$$

An open structure was modeled by taking the screening walls sufficiently far away from the guide. The results in the table are computed for the nondimensional normalized parameters Z and V, where

$$V = k_0 a_x \sqrt{\epsilon - 1} \qquad Z = \frac{\beta^2/k_0^2 - 1}{\epsilon - 1}.$$

For all values of α the agreement is excellent.

The last table (Table V) shows the convergence and speed of the IEEM FFT for an open cylindrical inhomogeneous guide with the permittivity given by (9) with $\alpha = 2$ and $a_x/a_y = 1$. The screening walls were placed at a distance $10a_x$ from the center. The CPU time given in the last column is for an Intel 80386 (25 MHz) based PC with a Weitek numerical coprocessor. It was calculated by dividing the total run time by the number of iterations; hence figures are influenced by the time of relatively slow (8 MHz) i/o operations.

VII. CONCLUSIONS

An efficient approach to the analysis of dielectric guides is presented. The technique, called the IEEM FFT, combines a recently proposed iterative scheme (IEEM) with the FFT, resulting in an efficient algorithm requiring $O(n \log_2 n)$ operations and $O(n)$ computer locations, with n denoting the number of variables used in approximating the unknown fields. In terms of computer storage and speed, the method seems to be superior to most known full-wave approaches, including recent formulations of the finite difference and finite element methods which use dense matrix solvers.

ACKNOWLEDGMENT

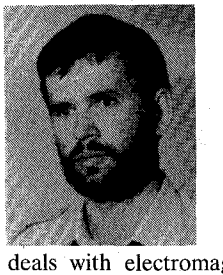
The author wishes to thank T. Jabłoński of the Polish Academy of Sciences for sharing his expertise on the IEEM and providing numerical data.

REFERENCES

[1] S. M. Saad, "Review of numerical methods for the analysis of arbitrarily shaped microwave and optical dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 894–899, Oct. 1985.
[2] K. Ogusu, "Numerical analysis of rectangular dielectric waveguide and its modifications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 874–885, Nov. 1977.
[3] W. Baier, "Waves and evanescent fields in rectangular waveguides filled with a transversely inhomogeneous dielectric," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 696–705, Oct. 1970.
[4] K. Bierwirth, N. Schultz, and F. Arndt, "Finite-difference analysis of rectangular dielectric waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1104–1114, Nov. 1986.
[5] T. F. Jabłoński, "Iterative eigenfunction expansion method for monomode gradient index fibres with arbitrary cross-section," in *Proc. URSI Int. Symp. E. M. Theory*, part B., 1986, pp. 415–417.
[6] T. F. Jabłoński and M. J. Sowiński, "Analysis of dielectric guiding structures by the iterative eigenfunction expansion method," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 63–70, Jan. 1989.
[7] M. Mrozowski, T. F. Jabłoński, and J. Mazur, "Application of the iterative eigenfunction expansion method to the analysis of rectangular dielectric waveguide structures," in *Proc. MIOP'90 Conf.* (Stuttgart), 1990, pp. 360–365.
[8] B. M. A. Rahman and J. B. Davies, "Finite-element analysis of optical and microwave waveguide problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 20–28, Jan. 1984.
[9] E. Schweig and W. B. Bridges, "Computer analysis of dielectric waveguides: A finite-difference method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 531–541, May 1984.
[10] T. F. Jabłoński, "Iterative eigenfunction expansion method for cylindrical fibres," (in Polish), *ITFR Reports*, 3/1986.
[11] B. S. Garbow, J. M. Boyle, J. J. Dongarra, and C. B. Moler, "Matrix eigensystem routines-EISPACK guide extension," in *Lecture Notes in Computer Science*, vol. 51. Berlin: Springer-Verlag, 1977.
[12] E. W. Kolk, N. H. G. Baken, and H. Blok, "Domain integral equation analysis of integrated optical channel and ridge waveguides in stratified media," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 78–85, Jan. 1990.
[13] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Sceintific Computing*. Cambridge: Cambridge University Press, 1986.
[14] J. J. Dongarra and E. Grosse, "Distribution of mathematical software via electronic mail," *Commun. Ass. Comput. Mach.*, vol. 30. pp. 403–407, May, 1987.
[15] T. F. Jabłoński, private communication.
[16] N. Schultz, K. Bierwirth, F. Arndt, and U. Köster, "Finite-difference method without spurious modes for the hybrid-mode analysis of diffused channel waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 722–729, June 1990.
[17] T. Angkaew, M. Matsuhara, and N. Kumagai, "Finite-element analysis of waveguide modes: A novel approach that eliminates spurious modes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 117–123, Feb. 1987.
[18] K. Hayata, K. Miura, and M. Koshiba, "Finite-element formulation for lossy waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 268–275, Feb. 1988.
[19] W. C. Chew and M. A. Nasir, "A variational analysis of anisotropic, inhomogeneous dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 661–668, Apr. 1989.

[20] J. A. M. Svedin, "A numerically efficient finite-element formulation for general waveguide problem without spurious modes," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1708–1715, Nov. 1989.

✠



Michał P. Mrozowski was born in Gdańsk, Poland, on May 12, 1959. He received the M.S. degree in electrical communication engineering and the Ph.D. degree in electronic engineering, both with first class honors, from the Technical University of Gdańsk in 1983 and 1990, respectively.

From 1984 to 1986 he was a Research Assistant at the Polish Academy of Sciences. In September 1986 he joined the Telecommunication Institute at the Technical University of Gdańsk, where he is now an Assistant Professor. From 1988 to 1989, under the British Council Fellowship Scheme, he was a visitor to University College London, where he worked on the theory of complex waves in lossless dielectric structures. His research deals with electromagnetic field theory and the development of methods for solving the problems of wave propagation in inhomogeneous isotropic and anisotropic media. His current work focuses on the fast numerical techniques for solving 2-D and 3-D boundary value problems and the investigation of novel effects in guides comprising anisotropic media. Dr. Mrozowski is a member of the Electromagnetics Academy.